AC-feedback electrostatic voltmeter measurements
Optimal measurement conditions

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Abstract An AC-feedback electrostatic voltmeter (ESVM) is a convenient tool for surface charge/voltage measurements. This paper describes a set of conditions which are required to achieve maximum accuracy for tests conducted with an AC-feedback ESVM.

1 Introduction

The AC-feedback electrostatic voltmeter (ESVM) is an example of Trek's original, advanced technology and expertise in non-contact surface voltage/charge measurements (U.S. patent no. 4797620, [1]). This instrument combines accuracy of the DC-feedback ESVM with convenience and low cost of an electrostatic fieldmeter. AC-feedback technology allows the voltmeter to be spacing independent within a specified range of probe-to-surface distances. Due to the very same AC-feedback, though, there exists a constraint regarding the type of surfaces that can be examined with accuracy specified by the manufacturer. Figure 1 illustrates an ESVM probe during measurements. The body of the probe and the sensor are capacitively coupled to the surface under test and to the surrounding environment.

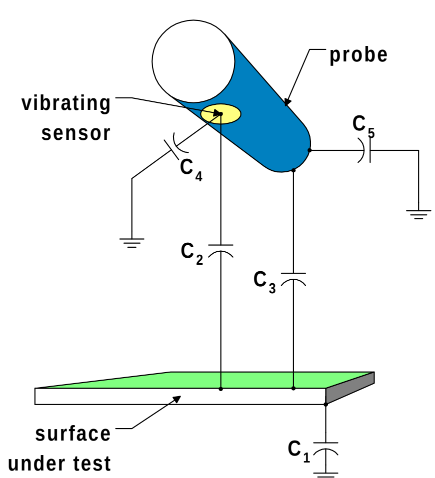

Figure 1: Probe and its capacitive couplings.

- C_1 is a capacitance between the surface under test and earth ground;
- C_2 is a capacitance between the probe sensor and the surface under test;
- C_3 is a capacitance between the probe body (enclosure of the probe) and the surface under test;
- C_4 is a capacitance between the probe sensor and the earth ground;
- C_5 is a capacitance between the probe body and the earth ground.

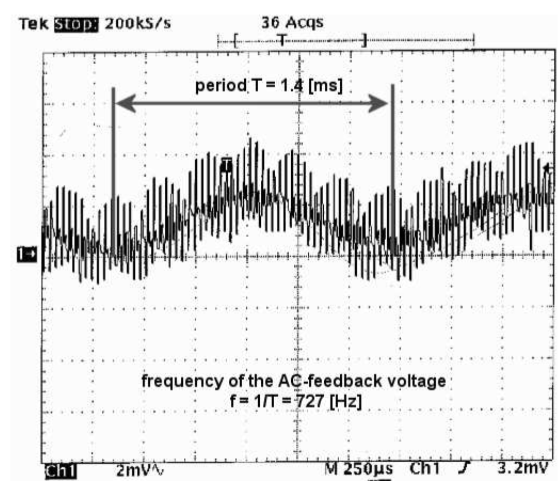

Figure 2: Oscilloscope picture of the AC-feedback induced on the test plate.

Ideally, during tests, the capacitance C_1 between the surface under investigation and ground is relatively high comparing to the body of the probe-to-ground capacitance C_5. However, if C_1 becomes comparable to C_5 the AC-feedback voltmeter readings become inaccurate. The AC feedback signal used by the voltmeter (see following sections of this application note for further explanation) is also being induced on the surface under test due to the capacitive coupling C_3 and C_2 (Figure 2). It creates an error offset voltage, which disturbs the measurement. Additionally the presence of a noise pick-up component can be noticed on the oscillogram. An intention of this application note is to describe the mechanism and consequences of the aforementioned coupling phenomenon.

2 AC-feedback operation

The AC-feedback voltmeter uses an original, innovative technique to achieve spacing independent surface voltage/charge measurements [1–3]. Rather than cancelling the Kelvin current I by use of a feedback DC voltage which follows the surface test voltage to produce zero electric field [4], the AC feedback method utilizes a nullifying current I' to zero the Kelvin current I. The current I' is produced by an internal generator circuit tuned to the frequency of the Kelvin sensor oscillations:

$$I' = C \cdot \frac{dV_t}{dt} \tag{1}$$

Therefore, when currents I and I' cancel each other, the following formula becomes valid:

$$\begin{aligned} I' &= I \\ U \cdot \frac{dC}{dt} &= C \cdot \frac{dV_t}{dt} \end{aligned} \tag{2}$$

Where:

U is the voltage on the surface under test,

$\boldsymbol{V_t}$ is the AC-feedback voltage supplied to the sensor in order to generate current I',

$\boldsymbol{C = C_2}$ is the capacitance between the sensor and the tested surface.

As both I and I' currents are inversely proportional to spacing D_0, the ratio of the amplitude of V_t to U (the DC test surface voltage) remains constant over the large range of D_0. Introduction of the additional AC signal on the surface under test will result in disturbance in the $U \cdot \frac{dC}{dt}$ component as U also becomes a function of time.

3 Theoretical model

Consider a model of an AC-feedback voltmeter as presented in Figure 1. The goal is to show the influence of the AC-feedback voltage u_2 on the voltage u_1 that is present on the tested surface. Figure 3 represents a corresponding circuit.

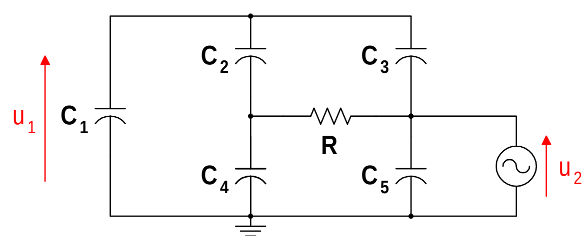

Figure 3: Model

R is a resistor across which the sensor current signal is detected. This current signal is actually a representation of the voltage on the surface under test (see the principle of operation of an AC feedback (ESVM);

$\boldsymbol{u_1}$ is the voltage on the surface under test, intended to be measured with the sensor. This voltage is the initial potential on the surface under test, referred to the earth ground.

$\boldsymbol{u_2}$ is an AC voltage fed back to the sensor and the body of the probe to compensate for the changes of distance between the surface and the sensor (from the principle of operation of an AC feedback ESVM).

This circuit can be solved using the principle of superposition. Therefore it has been split into 2 circuits, as presented in Figures 4 and 5. All reactances and resistances were, for simplicity, replaced by equivalent impedance elements.

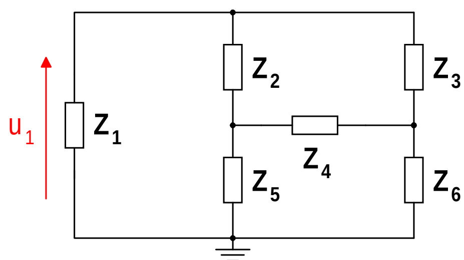

Figure 4: Superposition: model with voltage u_1.

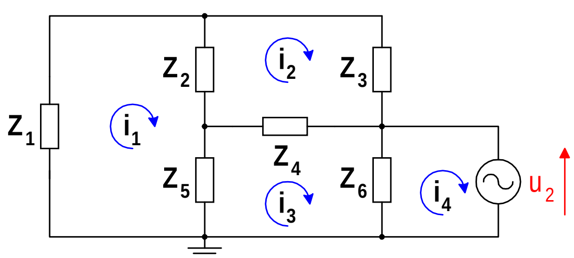

Figure 5: Superposition: model with voltage source u_2.

The voltage induced on capacitor C_1 with u_2 acting as the only voltage source in the circuit is of particular interest. In order to simplify further calculations, consider the circuit as shown in Figure 5. Mesh currents i_1 through i_4 are represented by the following system of equations:

$$\begin{aligned} i_1 \cdot Z_1 + (i_1\text{-}i_2) \cdot Z_2 + (i_1\text{-}i_3) \cdot Z_5 &= 0 \quad (3)\\ i_2 \cdot Z_3 + (i_2\text{-}i_3) \cdot Z_4 + (i_2\text{-}i_1) \cdot Z_2 &= 0 \quad (4)\\ (i_3\text{-}i_1) \cdot Z_5 + (i_3\text{-}i_2) \cdot Z_4 + (i_3\text{-}i_4) \cdot Z_6 &= 0 \quad (5)\\ (i_4\text{-}i_3) \cdot Z_6 &= u_2 \quad (6) \end{aligned}$$

Lets group expressions in equations by currents and use a matrix form. Define a matrix of impedances:

$$\mathbf{Z} = \begin{pmatrix} (Z_1+Z_2+Z_5) & \text{-}Z_2 & \text{-}Z_5 & 0 \\ \text{-}Z_2 & (Z_3+Z_4+Z_2) & \text{-}Z_4 & 0 \\ \text{-}Z_5 & \text{-}Z_4 & (Z_4+Z_5+Z_6) & \text{-}Z_6 \\ 0 & 0 & \text{-}Z_6 & Z_6 \end{pmatrix}$$

Followed by matrix of voltages:

$$\mathbf{U} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ u_2 \end{pmatrix}$$

$\mathbf{I}$ is a current matrix:

$$\mathbf{I} = \begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix}$$

System of equations 3 is now expressed in the form of:

$$\mathbf{Z} \cdot \mathbf{I} = \mathbf{U} \qquad (7)$$

Solving for current i_1:

$$i_1 = \frac{u_2 \cdot (Z_5Z_2 + Z_5Z_3 + Z_5Z_4 + Z_2Z_4)}{Z_5Z_1Z_2 + Z_5Z_4Z_3 + Z_5Z_1Z_3 + Z_5Z_1Z_4 + Z_5Z_2Z_3 + Z_1Z_4Z_3 + Z_2Z_1Z_4 + Z_2Z_4Z_3} \qquad (8)$$

Meanwhile, lets make assumptions about sensor-to-surface capacitance C_2 and the AC feedback voltage u_2. C_2 is vibrated by the tuning fork and is changing sinusoidally:

$$C_2 = \frac{\varepsilon \cdot S}{d_0 + d_1 \cdot \sin(\omega_1 t)} \qquad (9)$$

Voltage u_2 is also a sinusoidal signal:

$$u_2 = u_{20} \cdot \sin(\omega_2 t) \qquad (10)$$

Where:

ε - dielectric permittivity,

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S - surface area of the probe sensor,

$\boldsymbol{d_0}$ **-** sensor-to-surface distance when the sensor does not vibrate,

$\boldsymbol{d_1}$ **-** amplitude of vibrations of the sensor,

$\boldsymbol{\omega_1}$ **-** circular frequency of vibrations of the sensor,

$\boldsymbol{u_{20}}$ **-** amplitude of the voltage u_2,

$\boldsymbol{\omega_2}$ **-** circular frequency of u_2.

Define:

$$a = \frac{d_1}{d_0} \tag{11}$$

as a depth of modulation for vibrating sensor. Therefore:

$$\begin{aligned} C_2 &= \frac{\varepsilon \cdot S}{d_0 \cdot [1 + a \cdot \sin(\omega_1 t)]} = \\ &= \frac{C_{2_0}}{1 + a \cdot \sin(\omega_1 t)} \end{aligned} \tag{12}$$

Where:

$$C_{2_0} = \frac{\varepsilon \cdot S}{d_0} \tag{13}$$

Assume that the sensor-to-surface distance d_0 is much greater than the amplitude of sensor vibrations:

$$d_0 >> d_1 \Rightarrow a << 1, \tag{14}$$

It follows that:

$$C_2 \approx C_{2_0} \tag{15}$$

Thus C_2 can be considered constant, independent of ω_1.
With a sensor area S equal to $\approx 4.5 \cdot 10^{-6}$ [m^2] (for example Trek's Model 520) and typical probe-to-surface distance d_0 = 5 to 25 [mm] the value of C_2 is around 1.5 to 8 [fF] (10^{-15} [F]). Capacitive coupling of the sensor to the earth ground C_4 is significantly smaller than C_2.
Let's estimate also a capacitance C_3 between the probe body and the surface under test. It can be calculated as a capacitance between a cylinder and a plate (where the cylinder is parallel to the plate):

$$C_3 = \frac{2\pi\varepsilon}{acosh\left(\frac{d+R}{R}\right)} \cdot L \tag{16}$$

Where:

- d is a distance between the probe and the sensor;
- R is the radius of the probe;
- L is the length of the probe.

With typical values of d = 5to 25 [mm], L = 65.7 [mm], R = 5.5 [mm] (as it is, for example, in the case of 6000B-7C probe), capacitance C_3 is 3 [pF] for 5 [mm] and 1.5 [pF] for 25 [mm] distance, respectively. All above estimations are brought here to show that C_3 is significantly bigger (100 to 1000 times) than C_2.
Consider again equation 8. Let's substitute as follows (compare Figures 3 and 5):

$$\begin{array}{lll} Z_1 = -j\dfrac{1}{\omega_2 C_1} & Z_2 = -j\dfrac{1}{\omega_2 C_2} & Z_3 = -j\dfrac{1}{\omega_2 C_3} \\ Z_4 = R & Z_5 = -j\dfrac{1}{\omega_2 C_4} & Z_6 = -j\dfrac{1}{\omega_2 C_5} \end{array}$$

Therefore equation 8 becomes:

$$i_1 = \frac{u_2\omega_2 C_1(C_3 + C_2 + jR\omega_2 C_2 C_3 + jR\omega_2 C_4 C_3)}{-jC_3 + R\omega_2 C_1 C_2 - jC_2 + R\omega_2 C_2 C_3 - jC_1 + R\omega_2 C_4 C_2 + R\omega_2 C_4 C_3 + R\omega_2 C_4 C_1} \tag{17}$$

In order to simplify equation 17 it is convenient to use aforementioned assumptions about C_2 and C_4. If both those capacitances are negligibly small when compared to C_3 and C_1, it is possible to express i_1 as its approximation, i_1':

$$\begin{aligned} i_1' &= \lim_{\substack{C_2 \to 0 \\ C_4 \to 0}} i_1 \\ i_1' &= j\frac{u_2 C_3 C_1 \omega_2}{C_3 + C_1} \end{aligned}$$

And calculate u_1' as a voltage drop on Z_1:

$$\begin{aligned} u_1' &= Z_1 \cdot i_1' = \\ &= -j\frac{1}{\omega_2 C_1} \cdot i_1' \\ u_1' &= u_2 \cdot \frac{C_3}{C_3 + C_1} \qquad (18) \end{aligned}$$

Substituting equation 10 to formula 18:

$$u_1' = u_{20} \cdot \frac{C_3}{C_3 + C_1} \cdot \sin(\omega_2 t) \qquad (19)$$

By the superposition principle the voltage U_1 present on the surface under test is a sum of the voltage u_1 (which was originally present on the plate) and the voltage u_1' induced on the plate by the feedback voltage u_2.

$$U_1 = u_1 + u_{20} \cdot \frac{C_3}{C_3 + C_1} \cdot \sin(\omega_2 t) \qquad (20)$$

3.1 Error estimate

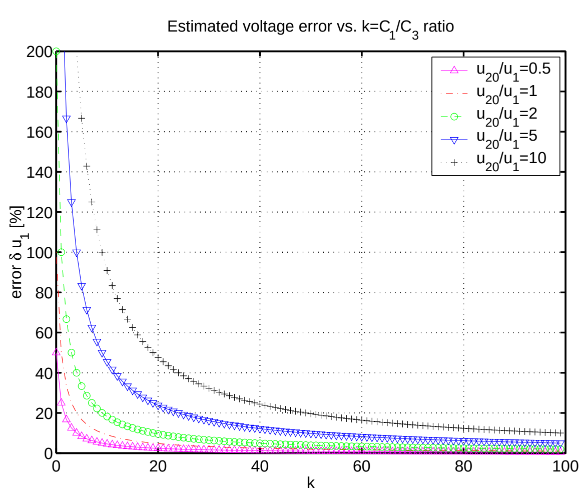

Figure 6: Estimated error

Let's estimate an error coming from the presence of u_1' on the surface under test. This systematic error Δu_1 is equal to:

$$\Delta u_1 = u_1' \qquad (21)$$

$$\Delta u_1 = u_{20} \cdot \frac{C_3}{C_3 + C_1} \cdot \sin(\omega_2 t) \qquad (22)$$

Let's make additional assumptions: first, the absolute value of the sine functions $\sin(\omega_2 t)$ is never greater than 1. Assume then that $u_{20} \cdot \sin(\omega_2 t) \approx u_{20}$.
Assume further that $C_1/C_3 = k$, so expression for Δu_1 becomes:

$$\Delta u_1 = u_{20} \cdot \frac{1}{1+k} \qquad (23)$$

Percentage error can be then shown as:

$$\begin{aligned} \delta u_1 &= \frac{\Delta u_1}{u_1} \cdot 100\% \\ \delta u_1 &= \frac{u_{20}}{u_1} \cdot \frac{1}{1+k} \cdot 100\% \qquad (24) \end{aligned}$$

With ratio u_{20}/u_1 kept constant, we receive the dependency between k and percentage error δu_1

4 Summary

It has been shown that to maintain appropriate accuracy of measurements with AC-feedback voltmeter it is necessary to provide sufficient capacitance between the surface under test and ground. Table 1 presents percentage errors along with respective capacitance ratios – all those values were obtained empirically. For example: if a typical Trek's side-view probe (round body, ϕ=8.7 [mm], length=48 [mm]) is placed 5 [mm] from the surface under test, its capacitive coupling to the surface (C_2) is around 2 [pF]. In order to achieve a measurement error of less than 5%, the surface to ground capacitance has to be greater than 19·2 [pF]=38 [pF].

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Error of measurement	$\frac{C_1}{C_3}$ ratio value
5%	19.0
4%	24.0
3%	32.3
2%	49.0
1%	99.0

Table 1: Error of measurement due to capacitance ratio.

References

[1] B. T. Williams. High voltage electrostatic surface potential monitoring system using low voltage a.c. feedback. U. S. patent no. 4797620, 1989.

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